

Prelim Exam

Each problem counts 30 points. You have 75 minutes. Please show your work. Good luck!

1. Consider the following two games: Answer the following questions for

	<i>L</i>	<i>C</i>	<i>R</i>
<i>T</i>	5, 2	3, 0	0, 1
<i>M</i>	4, 4	1, 4	1, 6
<i>B</i>	4, 1	0, 0	2, 2

this game. In (a) through (c), do not consider mixed strategies. **Pure strategies only.**

- (a) For each player, which strategies are weakly dominated?
- (b) For the row player, which strategies are strictly dominated?
- (c) What is the set of (pure strategy) Nash equilibria?
- (d) Find all the mixed equilibria.

Answer:

- (a) none.
- (b) *C* is strictly dominated by *R*.
- (c) *T, L* and *B, R*.
- (d) $(1/2, 0, 1/2)$ for row and $(2/3, 0, 1/3)$ for column.

2. Each of two people has 1 unit of a resource (time, energy, wealth, whatever) to allocate between fighting the other person and production. If each individual devotes y_i to fighting, then the total output produced is $2 - y_1 - y_2$ and person i obtains the fraction $p_i(y_1, y_2)$ of total output, where

$$p_i(y_1, y_2) = \begin{cases} 1, & \text{if } y_i > y_j, \\ 1/2, & \text{if } y_i = y_j, \\ 0, & \text{if } y_i < y_j. \end{cases}$$

Each person cares about the amount of output she receives, and wants to receive as much as possible.

- (a) Formulate this situation as a strategic form. (What are the strategy sets, the payoff functions, etc.)
- (b) Show that, in any Nash equilibrium, $y_1 = y_2$.
- (c) Find the set of all Nash equilibria.

Answer:

- (a) $\mathcal{N} = \{1, 2\}$. $A_1 = A_2 = [0, 1]$.

$$u_i(y_i, y_j) = (2 - y_i - y_j)p_i(y_i, y_j).$$

- (b) If $y_i < y_j$, then y_j can increase her earnings by choosing $y_i < y' < y_j$. You might have argued that i should jump ahead, but he cannot if $y_j = 1$, and jumping to 1 in this case would not improve him.
 - (c) If $y_i = y_j < 1$, then either player has a profitable deviation by increasing his or her y slightly. If $y_i = y_j = 1$, then there is no gain to anyone's lowering their y , so evidently this is the only Nash equilibrium.
3. An employer is planning to promote one employee from among three workers in a department. She will run a tournament to decide whom to promote. She will observe everyone's work effort for a month and reward the employee who works the hardest. In case of a tie she will choose randomly from among the winners. Each employee can put out either normal effort (N) or extra effort (E). Getting promoted has a utility of 10, not being promoted gives a utility benefit of 0. The cost of working hard is 5, while that of working normally is 0. Payoff is benefit less cost.
- (a) Describe this situation as a normal form game?
 - (b) Find all the pure strategy equilibria.
 - (c) Find all the symmetric mixed strategy equilibria.

Answer:

(a) $\mathcal{N} = \{1, 2, 3\}$. $A_1 = A_2 = A_3 = \{N, E\}$.

$$u_i(a_1, a_2, a_3) = \begin{cases} 0 & \text{if } a_i = N \text{ and } a_{-i} \neq (N, N); \\ 10/3 & \text{if } a_i = N \text{ and } a_{-i} = (N, N); \\ 10/k - 5 & \text{if } a_i = E \text{ and } \#\{a_j : a_j = E\} = k; \end{cases}$$

- (b) Any 1 individual choosing E and the rest N is a NE. A deviation from an N gives a return of 0. Any 2 individuals choosing E is also a Nash equilibrium.
- (c) Suppose that each individual plays E with probability p . The expected return to E for individual i is

$$\begin{aligned} N : & \quad (10/3)(1-p)^2 \\ E : & \quad 10(1-p)^2 + 10p(1-p) + (10/3)p^2 \end{aligned}$$

Individual i is indifferent between N and E if and only if $p = 1/2$.