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Heckscher–Ohlin Trade Theory

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Abstract

Heckscher–Ohlin trade theory consists of four principal theorems, viz. the Heckscher–Ohlin trade theorem whereby relatively capital-abundant countries export relatively capital-intensive commodities, the factor-price equalization theorem whereby trade in goods may serve to equalize wage rates between countries, the Stolper–Samuelson theorem whereby an increase in the price of the relatively labour-intensive commodity unambiguously improves the real wage rate, and the Rybczynski theorem stating that an increase in capital endowment by itself must cause some output to fall if prices are held constant. The article discusses the nature and fate of these theorems.

Keywords

Autarky; Comparative advantage; Dimensionality; Distribution of income; Factor- intensity reversal; Factor-price equalization theorem; Free trade; General equilibrium; Heckscher–Ohlin theorem; Hecksher–Ohlin trade theory; International trade theory; Labour–capital ratio; Leontief paradox; Metzler tariff paradox; New trade theory; Reciprocity relationship; Relative factor abundance; Relative factor intensity; Rybczynski theorem; Specialization; Stolper–Samuelson theorem; Tariffs; Terms of trade

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Eli Heckscher (1919) and Bertil Ohlin (1933) laid the groundwork for substantial developments in the theory of international trade by focusing on the relationships between the composition of countries' factor endowments and commodity trade patterns as well as the consequences of free trade for the functional distribution of income within countries. From the outset general equilibrium forms of analysis were utilized in these developments, which gradually came to be sorted out into four 'core propositions' (Ethier 1974) in the pure theory of international trade.

The Four Theorems

Although all four of the propositions to be discussed are an outgrowth of the seminal work of Heckscher and Ohlin, only one of these propositions bears their name explicitly. The *Heckscher–Ohlin theorem* states that countries export those commodities which require, for their production, relatively intensive use of those productive factors found locally in relative abundance. The twin concepts of relative factor intensity and relative factor abundance are most easily defined in the small dimensional context in which the basic theory is usually developed. Two countries are engaged in free trade with each producing the same pair of commodities in a purely competitive setting, supported by constant returns to scale technology that is shared by both countries. Each commodity is produced separately with inputs of two factors of production that, in each country, are supplied perfectly inelastically. (For a throrough analysis of having endowments respond endogenously, see Findlay 1995). Following the Ricardian distinction, commodities are freely traded but productive factors are internationally immobile.

Although one country may possess a larger endowment of each factor than another, the presumed absence of returns to scale guarantees that only relative factor endowments are important. The home country is said to be relatively labour abundant if the ratio of its endowment of labour to that (say) of capital exceeds the corresponding proportion abroad. This is known as the physical version of relative factor abundance. An alternative involves a comparison of autarky relative factor prices in the two countries: the home country can be defined to be relatively labour- abundant if its wage rate (compared with capital rentals) is lower before trade than is the foreign wage (relative to foreign capital rentals). Since autarky factor prices are determined by demand as well as supply conditions, these two versions need not correspond. In particular, if the home country is, in the physical sense, relatively labour abundant it might nonetheless have its autarky wage rate relatively high if taste patterns at home are strongly biased towards the labour-intensive commodity compared with tastes abroad. In such a case the trade pattern reflects the autarky factor–price comparison: the home country exports the physically capital-intensive commodity. As discussed below, the link between commodity price ratios (the proximate determinant of trade flows) and factor price ratios is more direct than that between commodity price ratios and physical factor endowments. Thus the Heckscher–Ohlin theorem is more likely to hold if relative factor abundance is defined in terms of relative factor prices prevailing before trade. The procedure typically followed in the literature is to assume that both countries share identical and homothetic taste patterns. Such an assumption, in conjunction with the presumed identity of technology at home and abroad (with an even stronger version of homotheticity–linear homogeneity) helps to isolate the separate influence of physical factor supplies and makes the validity of the Heckscher–Ohlin theorem with the physical definition of factor abundance as likely as with the autarky factor price definition.

These assumptions are less than sufficient to guarantee the Heckscher–Ohlin theorem, even in the simple context of two-country, two-factor, two-commodity trade. The potential stumbling block is the fact that even though countries share the same technology, the commodity that is produced by relatively labour-intensive techniques at home may be produced by relatively capital-intensive techniques abroad. This is the phenomenon of factor-intensity reversal. If production processes are independent of each other, there is nothing (other than bald assumption) to rule out its appearance. The bald assumption would assert that regardless of factor endowments one industry always employs a relatively higher ratio of labour to capital than does the other industry, where techniques are chosen with reference to the wage/rental ratio common to both industries. If this is not the case, and if the commodity that is relatively labour-intensive at home is produced by relatively capital-intensive techniques abroad, the phrasing of the Heckscher–Ohlin theorem that explicitly states ‘each country exports the commodity that is produced in that country making relatively intensive use of the factor found in relative abundance in that country’ is fatally flawed. The reason? If the relatively labour-abundant country exports its labour-intensive commodity, it must do so in exchange for the commodity that, in the relatively capital-abundant foreign country, is produced by labour- intensive techniques. Thus if one country satisfies the theorem, the other country cannot (Jones 1956).

In the event of factor-intensity reversal, it must be the case that, whatever the commodity exported by the labour-abundant home country, the ratio of labour to capital employed in its production must exceed the labour/capital intensity adopted in foreign exports. However, this observation is of little value if one wishes to infer from an intensity comparison between exportables and import-competing goods within a given country whether that country is more labour abundant than some foreign country. Such an inference lay behind the celebrated study of Leontief (1953) on United States trade patterns. This research, the conclusions of which came to be known as the Leontief paradox (American exportables are produced by more labour- intensive techniques than are import-competing goods) provided the major stimulus to developing and defining the meaning and conditions supporting the Heckscher–Ohlin theorem.

Earlier work in Heckscher–Ohlin trade models was focused on the pricing relationships embodied in Heckscher–Ohlin theory. Ohlin (1933) stressed the effect which free trade would tend to have on the distribution of income within countries, viz. relative factor prices would move in the direction of equality between trading countries which share the same technology. Ohlin's mentor, Heckscher, went even further in his pioneering 1919 article. Absolute factor-price equalization was purported to be 'an inescapable consequence of trade' (For recent appraisals of each of these economists see Jones 2002, 2006a). Nonetheless, Ohlin's view of partial equalization seems to have dominated, with the exception of Lerner's unpublished 1933 manuscript (which surfaced after Samuelson's articles), until the statement of the factor-price equalization theorem in articles by Samuelson in 1948 and 1949. Rejecting his earlier tacit acceptance of the Ohlin thesis of partial equalization (in the Stolper–Samuelson article, which appeared in 1941), Samuelson proved that within the traditional confines of the $2 \times 2 \times 2$ model (with no factor-intensity reversals and each country incompletely specialized), free trade would drive wage rates to absolute equality in the two countries (and, as well, would equate returns to capital) despite the assumption that labour (and capital) are assumed to be immobile between countries.

The logic of the argument for the simple 2×2 case can be stated briefly. In a competitive equilibrium unit cost equals price if the commodity is produced. Thus let A represent the matrix of input–output coefficients, aij, w the vector (pair) of factor prices, and p the vector (pair) of commodity prices. Techniques need not be constant; in general they depend upon prevailing factor prices so that $A = A(w)$. Therefore the competitive profit conditions if both goods are actually produced dictate that:

$$A(w).w = p. \qquad (1)$$

If we assume no factor-intensity reversals, $A(w)$ is non-singular. Therefore if countries share the same technology and face the same pair of free-trade commodity prices, they must face exactly the same set of factor prices if each country produces both goods.

This approach may suggest that the crucial issue in the factor-price equalization argument is the unique dependence of factor price vector w on commodity price vector p, and an extensive literature has developed which focuses on this issue. In the 2×2 case uniqueness is a simple question – it depends on factor intensities differing between sectors and not reversing. But from the outset Samuelson pointed out that this was not the only issue. The question of uniqueness involves properties of technology alone, whereas under appropriate circumstances two countries in free trade will have factor prices equalized only if factor endowments are reasonably similar. For, if factor endowments are too dissimilar, it will be impossible for both countries to produce both commodities, in which case the equalities in (1) cannot universally hold.

These ideas can be made more precise by considering a concept due to McKenzie (1955), which Chipman (1966) called the 'cone of diversification'. For any factor price vector, w, there is determined a pair of techniques (labour/capital ratios) for the two commodities. Both factors can be fully employed only if the country's endowment vector is contained within the cone spanned by these techniques. Suppose two countries face a common free-trade commodity price vector, p, and that the commonly shared technology associates a unique factor price w corresponding to this p. Then if the endowment vectors of both countries lie within the cone of diversification, their factor prices must be equalized (McKenzie 1955).

Some seven years prior to Samuelson's first factor-price equalization essay there appeared the article by Stolper and Samuelson (1941), which must be ranked a classic not only for its discussion of what became known as the *Stolper–Samuelson theorem*, but because it is one of the first concrete developments of the ideas of Heckscher and Ohlin in the explicit format of a two-factor, two-commodity, general equilibrium model (This theorem became so widely cited that on the golden anniversary of its appearance a conference was held at Stolper's university, the University of Michigan. See Alan Deardorff and Robert Stern 1994). Their argument supposedly concerns the effect of protection on real wages, and in the course of the argument they assume that a tariff does not change the terms of trade so that locally import prices rise. Subsequently, in what has become known as the 'Metzler tariff paradox', Metzler (1949) showed that with sufficiently inelastic demand a tariff might so improve a country's terms of trade that the relative domestic price of imports falls. If so, the Stolper–Samuelson contention that a tariff yields an increase in the real return to a country's relatively scarce factor would be reversed. However, it is now commonly agreed that the Stolper–Samuelson theorem refers to the general phenomenon whereby an increase in the relative domestic price of a commodity (whether brought about by a tariff increase, decrease, or some other reason) must unambiguously raise the real return to the factor of production used relatively intensively in the production of that commodity.

Introducing the production-box diagram technique (for a single country), Stolper and Samuelson illustrate how an increase in the relative price of labour-intensive watches attracts resources from capital-intensive wheat. To clear factor markets, both sectors must then use labour more sparingly. That is, the ratio of capital to labour utilized in each sector rises, which implies an unambiguous increase in labour's marginal productivity measured either in watches or in wheat. Thus regardless of workers' taste pattern, protection has increased the real wage.

The logic of the Stolper–Samuelson argument rests heavily upon the presumed absence of joint production. It takes labour and capital to produce watches, and, in a separate activity, a higher capital/labour ratio is used to produce wheat. In competitive settings any change in a commodity's price must reflect an average of factor price changes so that unit costs change as much as do prices. Therefore one factor price must rise relatively more than either commodity price. Which factor gains depends only upon the factor-intensity ranking. If the price of watches rises, and that of wheat does not, the wage rate must increase by relatively more. And this result follows even if techniques are frozen so that no resources can be transferred between sectors (as they can be in the Stolper–Samuelson discussion) and marginal products are not well defined (Jones 1965).

To round out the quartet of theorems, the Rybczynski theorem (1955) deals with the same model but focuses on the relationship between factor endowments and commodity outputs. Suppose commodity prices are kept fixed in the 2×2 setting and an economy is incompletely specialized. Then by the factor-price equalization theorem, factor prices are determined and fixed as well, which implies also that techniques of production remain constant. If the economy's endowment of one factor increases, while its endowment of the other factor remains constant, the economy must in some sense grow (the transformation schedule shifts out). However, this growth is strongly asymmetric: one output actually falls. The factor-intensity ranking selects the loser – the commodity that uses intensively the factor that is fixed in overall supply must decline. The reasoning is simple. As one factor expands, it must be absorbed in producing the commodity using it intensively. But with techniques frozen (since prices are assumed fixed), the expanding sector must be supplied with doses of the non-expanding factor as well. The only source for this factor is the other industry that must, perforce, contract.

Relationships Among the Theorems

All four propositions are based on the same 'mini-Walrasian' general equilibrium model of trade and there are some interesting relationships and distinctions among them. Perhaps most importantly, both the Heckscher–Ohlin theorem and the factor-price equalization theorem refer explicitly to a comparison between (two) countries, whereas the Stolper–Samuelson and Rybczynski propositions are involved with relationships within a single country. This distinction implies that the assumption that countries share an identical technology is not necessary for the latter two propositions. Thus, for example, a country could protect the factor used intensively in its import-competing sector in real terms (according to Stolper and Samuelson) regardless of the level or type of technology adopted by other countries.

The factor-price equalization theorem is a razor's-edge type of result. Should the technology available to two countries differ only slightly, any presumption of exact factor-price equalization in the absence of explicit international factor markets disappears. The Heckscher–Ohlin theorem is a little more robust in this regard. In general, trade patterns depend on all those variables that influence prices: tastes, technology, and factor endowments (not to mention taxes or other distortions). If tastes are identical (and homothetic) but factor endowments are not, the latter difference will tend to dominate the trading pattern even if technologies differ as long as this difference is 'less important'. At issue is a weighing of endowment differences with the Ricardian emphasis on technology differences. A particular variation of the factor-price equalization theorem is more general, and does not need to assume that technologies are identical between countries. It concerns the *dependence* of factor prices only upon commodity prices. It follows as long as the country produces both commodities.

Two versions of the Heckscher–Ohlin theorem have been cited, depending on which definition of relative factor abundance is selected. If the physical factor intensity ranking is chosen as the criterion, the basis for the Heckscher–Ohlin theorem resides in the kind of link between endowment patterns and outputs for a single economy exemplified by the Rybczynski theorem. An extension of this theorem allows a comparison of the transformation schedules for two economies with similar technologies. The relatively labour abundant (physical definition) country will produce relatively more of the labour-intensive commodity at common commodity prices (Jones 1956). Therefore, unless taste differences are sufficiently biased to counter this effect, the labour-intensive good will, in autarky, be cheaper in the labour-abundant country and, with trade, will be exported. The Stolper–Samuelson theorem is closely linked to the alternative form of the Heckscher–Ohlin theorem. Suppose there are no factor-intensity reversals. Then if both goods are produced there is a monotonic relationship between the wage/rent ratio and the relative price of the labour-intensive good such that a rise in the latter is associated with a greater than proportionate increase in the former. Thus the relatively low wage country must, in autarky, have been the relatively cheap producer of the labour-intensive commodity. As mentioned earlier, no caveat must be added about tastes, since these are already incorporated in the autarky factor-price comparison.

Although a comparison of factor endowments between countries is crucial in considering both the Heckscher–Ohlin theorem and the factor-price equalization theorem, such a comparison works in opposite directions for these two propositions. Thus if factor endowment proportions are sufficiently *dissimilar*, trade patterns suggested by the Heckscher–Ohlin theorem *must* hold (aside from the possibility of factor-intensity reversals) whereas free trade *cannot* bring about factor-price equalization. Sufficiently different factor endowments entail one country's transformation schedule being everywhere flatter than the other country's. At least one country must be specialized with trade. By contrast, the factor-price equalization result holds if factor endowments are similar enough so that international differences in the composition of outputs are capable of absorbing these endowment differences at the same set of techniques (and factor prices). If endowments are this close, it would always be possible for demand differences to be so biased that the physically labour-abundant country exports the capital-intensive commodity. Indeed, if such a demand reversal of the Heckscher–Ohlin theorem takes place, free trade must result in factor-price equalization (Minabe 1966).

Samuelson's name occurs so frequently in the literature on Heckscher–Ohlin trade theory that it is often appended to the other two names. One of his results not cited heretofore is the reciprocity relationship (Samuelson 1953). This states that in any general equilibrium model the effect of an increase in a commodity price (say p_j) on a factor return (say w_i) is the same as the effect of an increase in the corresponding factor endowment (V_i) on the output of commodity j. Of course, in each case some other set of variables is being held constant. Thus:

$$\frac{\partial w_i}{\partial p_j} = \frac{\partial x_j}{\partial V_i} \tag{2}$$

with all other commodity prices and all endowments held constant in the left-hand derivative and all other endowments and all commodity prices held constant in the right-hand derivative. This relationship is easy to prove (see, for example, Jones and Jose Scheinkman 1977). It also reveals the *dual* nature of the Stolper–Samuelson and Rybczynski theorems. If an increase in the price of watches lowers capital returns, then an increase in the endowment of capital (at constant prices) would lower the output of watches. In each case it is the presumed labour-intensity of watches that is operative.

In the 2 × 2 setting both the Stolper–Samuelson and Rybczynski theorems reflect the 'magnification effects' (Jones 1965) that stem directly from the assumed lack of joint production. With a '^' over a variable designating relative changes, if watches are labour intensive and wheat capital intensive and if the relative price of watches rises,

$$\widehat{w} > \widehat{p}_{wa} > \widehat{p}_{wh} > \widehat{r}. \tag{3}$$

In addition, should an economy grow, but with labour (L) growing more rapidly than capital (K),

$$\widehat{x}_{wa} > \widehat{L} > \widehat{K} > \widehat{x}_{wh}. \tag{4}$$

Inequality ranking (3) shows commodity price changes trapped between factor-price changes (since two factors are required to make a single good), while inequality (4) shows that in order to absorb endowment changes, the composition of outputs (each of which uses both factors) must change more drastically. Stolper and Samuelson stressed the first inequality in (3), while Rybczynski focused on the last inequality in (4), assuming $\widehat{K}$ equals zero.

Higher Dimensions

International trade theory generally, and Heckscher–Ohlin trade theory in particular, has frequently been criticized for its restriction to the low dimensionality represented by two commodities, two factors, and two countries. In fairness to both Heckscher and Ohlin it should be stressed that their discussions typically were not so confined. But neither were their conclusions as precise as those subsequently developed by Samuelson and others in the 2 × 2 × 2 versions of the four core propositions. And in the years following Samuelson's pioneering work on factor-price equalization, scores of articles have indeed appeared dedicated to the question of robustness of these results in higher-dimensional contexts. A highly detailed discussion of the issue of dimensionality is provided in Ethier (1984), and an earlier critique of the limitations imposed by small numbers of goods and factors is found in Ethier (1974) and Jones and Scheinkman (1977).

Part of the difficulty embedded in the move to higher dimensions lies in the ambiguity involved in what the propositions should state for cases beyond 2 × 2. The one proposition for which this is not the case is the factor-price equalization theorem. Consider the case of equal numbers of factors and produced commodities, with all goods traded and factors immobile internationally. The uniqueness of a factor price vector, *w*, corresponding to a given commodity price vector, *p*, is not guaranteed even in the 2 × 2 case; a factor-intensity reversal could lead to two (or more) values of *w* consistent with a given *p*. For the *n* × *n* case Gale and Nikaido (1965) provided conditions sufficient to guarantee global univalence of the factor price–commodity price relationship: the *A*(*w*) matrix of input–output coefficients should be a 'P- matrix', that is a matrix with all positive principal minors. These conditions have been slightly weakened by Andreu Mas-Colell (1979), and earlier a fundamental interpretation of the conditions was supplied by Yasuo Uekawa (1971). It remains the case, however, that this condition on technology alone is somewhat remote from the issue of factor-price equalization. Just as in the 2 × 2 case, two countries sharing a common technology and each capable of producing the same set of *n* commodities (at the same traded-goods prices) with *n* productive factors, will, if techniques of production are the same, have their factor prices driven to equality if their factor endowments are sufficiently close. The concept of the 'cone of diversification' within which both endowment vectors must lie for factor-price equalization is as meaningful and relevant in *n* dimensions as it is in two.

Although the factor-price equalization theorem has an unambiguous meaning in higher dimensions, it is a theorem that cannot be expected to hold if the number of productive factors exceeds the number of freely traded commodities. The reasoning is basic, and can be linked to Eq. (1). These competitive profit conditions supply *n* links between factor prices and traded commodity prices, where *n* is the number of traded commodities. If *r*, the number of factors, should exceed *n*, the relationships in Eq. (1) are insufficient in number to provide a solution for the vector *w* for given *p*. Other conditions are required, and these are provided by the full employment conditions, one for each productive factor. Thus a nation's endowment bundle, *V*, becomes a determining variable affecting factor prices that is additional to the commodity price vector, *p*. For example, in the simple three-factor, two-commodity 'specific-factor' model (Jones 1971; Samuelson 1971), suppose a country faces a given world price vector, *p*, and experiences a slight increase in its endowment of a factor 'specific' in its use in the first industry. The intensity with which factors are utilized depends upon factor prices, and if these do not change, there is no way in which outputs can adjust to clear all factor markets. The return to the factor specific to the first industry must fall so as to encourage the further use of that factor. Two countries of this type with different endowments will generally have different sets of factor prices with trade, even if they share a common technology. It may be interesting to note that Heckscher's (1919) discussion of the necessity of factor-price equalization is focused on a three-factor, two-commodity numerical illustration (Jones 2006a). As just suggested, such a 3 × 2 setting in general does *not* lead to factor-price equalization.

The Heckscher–Ohlin model with two factors but many commodities available in world markets provides a useful scenario in which to re-examine the Heckscher–Ohlin theorem concerning the pattern of trade. The strong influence of factor endowments on production and trading patterns is revealed by considering two countries sharing the same technology but with different endowment ratios. Suppose commodity prices for traded goods are determined in a world market composed of a number of different countries with potentially a wide variation in technologies. Given world prices, any pair of countries with the same technologies shares a Hicksian composite unit-value isoquant for all traded goods (Jones 1974), made up of strictly bowed-in sections (where only one commodity is produced) alternating with flats (where a pair of commodities is produced). Regardless of the number of commodities, each country engaged in trade need produce only one or two (in the two-factor case), and these commodities will be the ones requiring factors in proportions close to that country's endowment ratio (Not explicitly considered here is that the activity of exporting a commodity may require factor proportions different from those required in production; see Jones et al. 1999). In this setting the spirit of the Heckscher–Ohlin theorem is that each country concentrates its resources on a small range of commodities whose factor requirements mirror closely that country's endowment base; the country exports some or all commodities in this set and imports commodities that are more labour-intensive than these goods as well as those that are more capital-intensive. Two countries whose endowments are fairly similar *may* produce the same pair of goods and thus achieve factor-price equalization with trade. Countries further apart in endowment composition will have disparate sets of factor prices and may produce completely different bundles of commodities (see also Krueger 1977).

With many factors and many commodities a different approach can be taken. The ability of autarky commodity price comparisons to predict trade patterns item by item is severely questioned, so that little hope remains of linking endowment differences to the detailed composition of trade. But statements about aggregates or 'correlations' between trade patterns and autarky prices can be made (Deardorff 1980; Dixit and Norman 1980). A nation's net imports, M, are positively correlated with the comparison of its autarky commodity price vector, p^A, and the vector of free-trade commodity prices, p^T. Thus:

$$\left(p^A - p^T\right)M \geq 0 \tag{5}$$

(see Ethier 1984, p. 139). This idea can be extended to the further relationship between autarky commodity prices and the vector of autarky factor prices (as in (1)) to establish that countries possess a comparative advantage, on average, in commodities using intensively factors that are relatively cheap in autarky (See Deardorff 1982, and Ethier 1984, for more details).

The reciprocity relationship expressed in (2) is quite general in terms of dimensionality and thus serves to link the Rybczynski theorem in a dual relationship to the Stolper–Samuelson theorem. However, when the number of factors exceeds the number of produced commodities, differences between the two types of theorems do appear. This basic asymmetry is linked to the failure of the factor-price equalization theorem when factors exceed commodities in number.

Major efforts have been made to generalize the Stolper-Samuelson and Rybczynski theorems from the 2 × 2 settings to the $n \times n$ setting, and a pair of earlier efforts met with only limited success. Murray Kemp and Leon Wegge (1969) searched for conditions on the original activity matrix, A, or distributive share matrix, θ, that would be sufficient to ensure what is known as the *strong* form of the Stolper–Samuelson theorem: Each factor is associated with a unique commodity such that if that commodity price (alone) increases, the return to the associated factor increases by a relatively greater extent and all other factor returns fall. The conditions they tried are stated by the inequalities in (6):

$$\theta_{ii}/\theta_{ki} > \theta_{ij}/\theta_{kj} \quad \text{for all } i, j \neq i, \text{and } k \neq i. \tag{6}$$

For each factor, i, the ratio of its distributive share in the industry positively associated with that factor (i) to that of any other factor's share in industry i, exceeds the corresponding ratio of these two factors in any other industry. These strong conditions do indeed lead to the desired strong result on factor returns (that is the inverse of the distributive share matrix has positive diagonal terms, greater than unity, and negative off-diagonal elements) for the 3 × 3 case. However, the authors provided a counter-example for the 4 × 4 case and that was that. Even stronger conditions for sufficiency are required, and these were supplied by Jones et al. (1993). These conditions are, in a sense, suggested by the statement of the theorem that for any price change all factor returns except one must fall. That is, they must have a relatively similar fate, suggesting fairly similar intensity use. The inequality that suffices is shown in (7):

$$\theta_{ii}/\theta_{ki} - \theta_{ij}/\theta_{kj} > \Sigma_{s \neq k,i,j} |\theta_{si}/\theta_{ki} - \theta_{sj}/\theta_{kj}| \text{ for all } i; j, k \neq i. \quad (7)$$

That is, condition (6) is not strong enough; the difference between the two terms in (6) must exceed the absolute value of similar differences in all the unintensive factors (whose returns all must fall). As occasionally happens, an article by John Chipman (1969) in the same issue of the same journal provided a condition sufficient for a weaker result, namely that the elements along the diagonal all be positive and exceed unity, regardless of signs off the diagonal. His condition met the same fate – sufficient for the 3 × 3 case but not higher. Mitra and Jones (1999) provided a sufficient condition for the $n \times n$ case.

It is possible to argue that these conditions are so strong as to suggest the Stolper–Samuelson and Rybczynski theorems really do not generalize. However, there is a form of the Stolper–Samuelson theorem that does generalize to higher dimensions with relatively little structure and, arguably, captures the essence of the original 1941 result. Stolper and Samuelson addressed the question of a particular government policy on real wages – the imposition of a tariff. But consider a more general question. Suppose an arbitrary factor of production seeks government aid sufficient to have its *real return* improved in a *non-transparent* fashion, that is, without a direct payment (out of tariff or any other source of government revenue). It is to be done by changes in taxation or government demand that would affect commodity prices. What would be required? Very little, as shown in Jones (1985, 2006b). Suppose there is little or no joint production (to be discussed below), and that there is a sufficient number of commodities (at least equal to the number of factors). These conditions suffice to ensure that there exists a subset of commodities that, if their prices are raised by the same relative amount with no other commodity price changes, the real return to the particular factor is guaranteed to increase. This result should have pride of place in the field of political economy and represents a significant generalization of the original Stolper–Samuelson result. The kind of detailed requirements shown by (6) and (7) is not necessary as long as a single commodity is not by itself required to do the job and as long as (along with Chipman 1969) it is not required that all other factors lose. Indeed, the favoured factor might well appreciate not standing nakedly as the only winner.

There are a few special cases of the $n \times n$ Heckscher–Ohlin setting that deserve mention. The most important might be the contribution of Roy Ruffin (1988). Ruffin redefines the Ricardian setting in which each country has a distinct labour force whose productivities in producing a number of goods differ from those in other countries. Instead of having each type of labour restricted to a single country, Ruffin suggests letting each country be populated by a wide variety of labour types, with the relative supplies of each type differing from country to country. This shifts the focus to relative endowment differences among countries, with the same technologies (a single type of labour is the same no matter where located). The key feature of such a model is that there is not only no joint production of outputs, there is no need for any single factor to have to work jointly with any others to produce commodities. As a consequence, factor prices are always equalized by free trade in commodities. Furthermore, each country's transformation surface looks just like that of a *world* transformation surface in the Ricardian model. In the two-commodity case this is a broken, bowed out, join of the two linear schedules for each country. In higher dimensions there are various dimensional 'facets' down to those of zero-dimension, that is, points at which each type of labour is fully employed in a different commodity. Except for the relative size of these facets, each country's transformation curve looks like that of any other country. At given commodity prices the common 'price plane' is 'tangent' to each surface such that each labour type is assigned to the same commodities in any country. At free-trade prices the relative production pattern in any country exactly mirrors the relative labour supplies and productivity of labour in that country.

Another special version, one that does give the Kemp–Wegge strong results, is the 'produced mobile factor' structure introduced by Jones and

Marjit (1985, 1991). Imagine an $\{(n + 1) \times n\}$ specific-factors structure, with n specific factors and a single factor mobile between sectors. This is often taken to be labour, but instead, suppose it is a mobile input that is produced by all the specific factors. (That is, each 'specific' factor produces a particular commodity and, in addition, joins with other factors to produce the mobile factor.) This reduces to an $n \times n$ model with strong Stolper–Samuelson properties.

Fred Gruen and Max Corden (1970) introduced a simple model in discussing the possibility that a country such as Australia might, in levying a tariff, worsen its terms of trade. There are two sectors in the economy, manufacturing and agriculture. The manufacturing sector consists of a single commodity produced by labour and capital. Agriculture has two commodities, wheat and wool, each using labour and land. Thus, this is a special form of 3×3 model. As developed by Jones and Marjit (1992), it is possible to consider the $n \times n$ version of the Gruen–Corden model in which $(n - 2)$ sectors of the economy each use mobile labour and a type of capital specific to that sector to produce a single distinct commodity. In another sector labour and a specific type of capital produce a pair of commodities just as in the original Gruen–Corden case. An application of the Gruen and Corden model is also found in Findlay (1993).

The point of each of these special settings is that Heckscher–Ohlin models in the $n \times n$ case need not be difficult to analyse. However, the most popular two-factor model in the many commodity case may well be more valuable in that it focuses attention on which good or pair of goods a country produces in an international setting. Trade allows a great degree of specialization, and this version of the Heckscher–Ohlin model allows something that the special $\{n \times n\}$models, as well as the $\{(n + 1) \times n\}$ specific-factors model, do not, *viz*. treating the pattern of production as *endogenous* (see also Jones 2007a, b).

Joint Production

Both the Stolper–Samuelson theorem and the Rybczynski theorem are essentially reflections of the asymmetry between factors and commodities. This asymmetry is characterized by the assumption that productive activities are non-joint: in the non- degenerate cases more than one input is required to produce, separately, each output. Thus each commodity price change is a positive weighted average of the changes in rewards to factors used to produce that commodity. This implies that regardless of the ranking of commodity price changes, there is some factor reward that would rise relative to any commodity price rise and at least one factor reward which would rise by relatively less (or fall by more) than any commodity price change. Allowing joint production potentially destroys this asymmetry and thus the basis for the magnification effects.

There is a small literature dealing with this issue (Jones and Scheinkman 1977; Chang et al. 1980; Uekawa 1984). Much depends on the range of output proportions in any productive activity compared with the range of input proportions. For example, in the 2×2 case suppose one activity produces primarily the first commodity, but also a small amount of the second, while the other activity reverses these proportions. Furthermore, suppose this 'output' cone of diversification contains the standard 'input' cone of diversification. In this case traditional magnification effects underlying the Stolper–Samuelson and Rybczynski theorems remain valid. New results emerge if these cones intersect or the input cone contains the output cone (Cones can be made comparable by using distributive shares of inputs and outputs in activities).

Joint production does not, by itself, interfere with the status of the factor-price equalization theorem (Jones 1992; but see Samuelson 1992, for an alternative view). However, joint production does suggest an alteration of the Heckscher–Ohlin theorem. Instead of concentrating on the link between factor endowments and the location of commodity outputs (and therefore trading patterns), the focus is on the location of productive activities. Each activity requires, as before, an array of inputs, and the allocation of endowment bundles among countries helps to determine where these activities are located. The pattern of commodity trade must then reflect, as well, the output composition of these activities.

Concluding Remarks

The theory of international trade that has developed from the seminal writings of Heckscher and Ohlin is fundamentally based on the twin observations that countries differ from each other in the composition of their factor endowments and that productive activities are distinguished by the different relative intensities with which factors are required. As this theory has been developed four core propositions have served to summarize its content. The strict validity of each of these propositions has been seen to depend upon further specification of the technology (for example, ruling out factor intensity reversals, joint production, and non-constant returns to scale, and imposing, for some results, that countries share the same technology), demand (for example, requiring all individuals to possess identical homothetic taste patterns), or dimensionality (for example, requiring a small number of factors and commodities, or a matching number of both). To conclude this discussion of the core propositions it is possible to point out the less precise, broad message of each.

The Heckscher–Ohlin Theorem

Production patterns reflect different compositions of endowments and, unless demand differences are significant, so will patterns of trade. International trade encourages specialization in production in those activities requiring factors in proportions similar to the endowment bundle and allows a country to import commodities whose factor requirements are far from proportions found at home. In some of the writings on ‘new trade theory’, assumptions are made that all varieties of a certain type of product are produced using the same factor proportions. By assumption this rules out the Heckscher–Ohlin theorem as an explanation of trade patterns. However, if varieties differ in quality, each variety could differ in factor requirements as well, serving to re-establish the relevance of the Heckscher–Ohlin theorem.

The Factor-Price Equalization Theorem

Even if the international mobility of factors of production is ruled out by national frontiers, free trade in commodities helps to even out disparities in demand relative to supply of factors and to diminish the discrepancy between factor returns among countries. Two or more countries sharing the same technology will find that free trade brings factor returns to absolute equality if their endowments are sufficiently similar and they produce in common a sufficient number of commodities (at least equal to the number of distinct productive factors).

The Stolper–Samuelson Theorem

Changes in relative commodity prices, such as those brought about by trade or interferences in trade, have strong asymmetric effects on factor rewards. If no joint production prevails, some factors find their real rewards unambiguously raised and other rewards are unambiguously lowered by relative price changes. If, further, the number of factors equals the number of produced commodities, as in the original 2×2 setting, and production is non-joint, relative commodity price changes can be constructed which, without the aid of any direct subsidies, will raise the real reward of any particular factor regardless of its taste pattern.

The Rybczynski Theorem

Unbalanced growth in factor supplies tends, at given commodity prices, to lead to stronger asymmetric changes in outputs. If the numbers of factors and commodities are evenly matched and production is non-joint, this asymmetry entails that growth in some, but not all, factors (when commodity prices are given) serves to force an actual reduction in one or more outputs. By similar reasoning, differences in the composition of endowments among countries with similar technologies results in stronger asymmetries in production patterns when all face free trade commodity prices. If tastes are somewhat similar, these endowments differences are apt to support the trading patterns described by the Heckscher–Ohlin theorem.

See Also

- ▶ Factor Price Equalization (Historical Trends)
- ▶ General Equilibrium
- ▶ International Trade Theory
- ▶ Leontief Paradox

Bibliography

Chang, W., W. Ethier, and M. Kemp. 1980. The theorems of international trade with joint production. *Journal of International Economics* 10: 377–394.

Chipman, J. 1966. A survey of the theory of international trade, Part 3. *Econometrica* 34: 18–76.

Chipman, J. 1969. Factor price equalization and the Stolper–Samuelson theorem. *International Economic Review* 10: 399–406.

Deardorff, A. 1980. The general validity of the law of comparative advantage. *Journal of Political Economy* 88: 941–957.

Deardorff, A. 1982. The general validity of the Heckscher–Ohlin theorem. *American Economic Review* 72: 683–694.

Deardorff, A., and R. Stern, eds. 1994. *The Stolper–Samuelson theorem: A golden jubilee*. Ann Arbor: University of Michigan Press.

Dixit, A., and V. Norman. 1980. *Theory of international trade*. Cambridge: Cambridge University Press.

Ethier, W. 1974. Some of the theorems of international trade with many goods and factors. *Journal of International Economics* 4: 199–206.

Ethier, W. 1984. Higher dimensional issues in trade theory. In *Handbook of international economics*, ed. R. Jones and P. Kenen, vol. 1. Amsterdam: North-Holland.

Findlay, R. 1993. Wage dispersion, international trade and the services sector. In *Trade, growth and development: The role of politics and institutions*, ed. G. Hansson. London: Routledge.

Findlay, R. 1995. *Factor proportions and growth*. Cambridge, MA: MIT Press.

Flam, H., and M.J. Flanders, eds. 1991. *Heckscher–Ohlin trade theory.* Cambridge, MA: MIT Press.

Gale, D., and H. Nikaido. 1965. The Jacobian matrix and the global univalence of mappings. *Mathematische Annalen* 159: 81–93.

Gruen, F., and W. Corden. 1970. A tariff that worsens the terms of trade. In *Studies in international economics*, ed. I. McDougall and R. Snape. Amsterdam: North-Holland.

Heckscher, E. 1919. The effect of foreign trade on the distribution of income. *Ekonomisk Tidskriff*: 497–512. Translated as chapter 13 in American Economic Association, *Readings in the theory of international trade*. Philadelphia: Blakiston, 1949, and a new translation is provided in Flam and Flanders (1991).

Jones, R. 1956. Factor proportions and the Heckscher–Ohlin theorem. *Review of Economic Studies* 24: 1–10.

Jones, R. 1965. The structure of simple general equilibrium models. *Journal of Political Economy* 73: 557–572.

Jones, R. 1971. A three-factor model in theory, trade, and history. In *Trade, balance of payments, and growth*, ed. J.N. Bhagwati et al. Amsterdam: North-Holland.

Jones, R. 1974. The small country in a many-commodity world. *Australian Economic Papers* 13: 225–236.

Jones, R. 1985. Relative prices and real factor rewards: A reinterpretation. *Economic Letters* 19: 47–49.

Jones, R. 1992. Jointness in production and factor-price equalization. *Review of International Economics* 1: 10–18.

Jones, R. 2002. Heckscher–Ohlin trade models for the new century. In *Bertil Ohlin: A centennial celebration (1899–1999)*, ed. R. Findlay, L. Jonung, and M. Lindahl. Cambridge, MA: MIT Press.

Jones, R. 2003. Joint output and real wage rates. *International Review of Economics and Finance* 12: 513–516.

Jones, R. 2006a. Eli Heckscher and the holy trinity. In *Eli Heckscher, international trade, and economic history*, ed. R. Findlay et al. Cambridge, MA: MIT Press.

Jones, R. 2006b. 'Protection and real wages': The history of an idea. *Japanese Economic Review* 57: 457–466.

Jones, R. 2007a. Key international trade theorems and large shocks. *International Review of Economics and Finance*.

Jones, R. 2007b. Specific factors and Heckscher–Ohlin: An intertemporal blend. *Singapore Economic Review* 52: 1–6.

Jones, R., and S. Marjit. 1985. A simple production model with Stolper–Samuelson properties. *International Economic Review* 19: 565–567.

Jones, R., and S. Marjit. 1991. The Stolper–Samuelson theorem, the Leamer triangle, and the produced mobile factor structure. In *Trade, policy, and international adjustments*, ed. A. Takayama, M. Ohyama, and H. Otah. New York: Academic Press.

Jones, R., and S. Marjit. 1992. International trade and endogenous production structures. In *Economic theory and international trade: Essays in memoriam J. Trout Rader*, ed. W. Neuefeind and R. Riezman. Berlin/New York: Springer-Verlag.

Jones, R., and J. Scheinkman. 1977. The relevance of the two-sector production model in trade theory. *Journal of Political Economy* 85: 909–935.

Jones, R., S. Marjit, and T. Mitra. 1993. The Stolper–Samuelson theorem: Links to dominant diagonals. In *General equilibrium, growth and trade II: Essays in honor of Lionel W. McKenzie*, ed. R. Becker, R. Jones, and W. Thomson. New York: Academic Press.

Jones, R., H. Beladi, and S. Marjit. 1999. The three faces of factor intensities. *Journal of International Economics* 48: 413–420.

Kemp, M., and L. Wegge. 1969. On the relation between commodity prices and factor rewards. *International Economic Review* 10: 407–413.

Krueger, A. 1977. *Growth, distortion, and patterns of trade among many countries*, Princeton studies in international finance, no. 40. Princeton: Princeton University Press.

Leontief, W. 1953. Domestic production and foreign trade: The American capital position re-examined. *Proceedings of the American Philosophical Society* 97: 332–349.

Lerner, A. 1933. Factor prices and international trade. Mimeo. Published in *Economica* 19 (1952): 1–15.

Mas-Colell, A. 1979. Two propositions on the global univalence of systems of cost functions. In *General equilibrium, growth, and trade*, ed. J. Green and J. Scheinkman. New York: Academic Press.

McKenzie, L. 1955. Equality of factor prices in world trade. *Econometrica* 23: 239–257.

Metzler, L. 1949. Tariffs, the terms of trade, and the distribution of national income. *Journal of Political Economy* 57: 1–29.

Minabe, N. 1966. The Heckscher–Ohlin theorem, the Leontief paradox, and patterns of economic growth. *American Economic Review* 56: 1193–1211.

Mitra, T., and R. Jones. 1999. Factor shares and the Chipman condition. In *Trade, welfare and econometrics: Essays in honor of John S. Chipman*, ed. J. Melvin, J. Moore, and R. Riezman. New York: Routledge.

Ohlin, B. 1933. *Interregional and international trade*. Cambridge, MA: Harvard University Press.

Ruffin, R. 1988. The missing link: The Ricardian approach to the factor endowment theory of trade. *American Economic Review* 78: 759–772.

Rybczynski, T.M. 1955. Factor endowments and relative commodity prices. *Economica* 22: 336–341.

Samuelson, P. 1948. International trade and the equalization of factor prices. *Economic Journal* 58: 163–184.

Samuelson, P. 1949. International factor-price equalization once again. *Economic Journal* 59: 181–197.

Samuelson, P. 1953. Prices of factors and goods in general equilibrium. *Review of Economic Studies* 21: 1–20.

Samuelson, P. 1971. Ohlin was right. *Swedish Journal of Economics* 73: 365–384.

Samuelson, P. 1992. Factor-price equalization by trade in joint and non-joint production. *Review of International Economics* 1: 1–9.

Stolper, W., and P. Samuelson. 1941. Protection and real wages. *Review of Economic Studies* 9: 58–73; also in Deardorff and Stern (1994).

Uekawa, Y. 1971. Generalization of the Stolper–Samuelson theorem. *Econometrica* 39: 197–213.

Uekawa, Y. 1984. Some theorems of trade with joint production. *Journal of International Economics* 16: 319–333.