Linear Economic Models

1. Are the following matrices productive? Prove your answer.

$$A_1 = \begin{bmatrix} 0.6 & 0.2 & 0.1 \\ 0.3 & 0.2 & 0.4 \\ 0.2 & 0.4 & 0.3 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0.6 & 0.5 \\ 0.1 & 0.5 \end{bmatrix}.$$

2. Prove that A is productive if and only if $(I - A)^{-1}$ is non-negative.

3. Prove that if A is productive, then there is at least one column sum of A which is less than 1.

4. Let

$$A = \begin{bmatrix} 0.1 & 0.4 & 0.3 \\ 0.2 & 0.7 & 0.0 \\ 0.1 & 0.1 & 0.5 \end{bmatrix}$$

Suppose too that the vector of labor requirements is $(1, 1, 1)$. Compute competitive equilibrium prices for this economy.

5. Associated with any $n \times n$ non-negative matrix A is a graph G where there is a directed edge from i to j iff $a_{ij} > 0$. A matrix is said to be *irreducible* iff the graph is strongly connected; that is iff there is a path from any i to any j.

   (a) Show that a square matrix A is irreducible iff for each pair i and j there is an m such that the i, j'th element of A^m is positive. [Hint: What does it mean for an element of A^2 to be positive, in terms of the graph?]

   (b) Use the preceding fact to show that the assumption $a_0 \gg 0$ can be replaced with the assumptions $a_0 > 0$ and A is irreducible in the characterization of equilibrium in the simple Leontief model.