Problem Set on Matchings:
TU Matching

1. Suppose individual's valuations for houses are as follows: Find the optimal match, and

		Houses		
		A	B	C
	X	5	7	1
Workers	Y	2	3	1
	Z	5	4	4

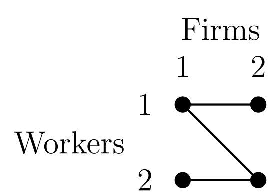
prices for individuals and houses.

		Houses		
		A	B	C
	X	10	θ	4
Workers	Y	7	5	3
	Z	4	3	2

2. For each $\theta > 5$, valuations increase in workers from bottom to top, for each firm, and valuations increase in firms from right to left for each worker. For what values of θ is the optimal match assortative?

3. Suppose that there are three populations: Low-skilled workers $l \in \mathcal{L}$, high-skilled workers $h \in \mathcal{H}$, and robots $r \in \mathcal{R}$. It takes one of each to to generate surplus: v_{lhr} is the surplus from the triple lhr. Define a competitive equilibrium, conjecture a relationship between competitive and optimal matches, and prove your conjecture.

4. Suppose we are given workers $\mathcal{L}$ and firms $\mathcal{F}$, and a set of edges $\mathcal{E}$ connecting workers with firms. This is an example of a bi-partite graph. The edges describe the set of feasible matches, so that not all matches are possible. Here is an example: Worker 1 can



match with both firms, but worker 2 can match only with firm 2. Denote by $\delta(l)$ and $\delta(v)$ the set of firms (workers) worker l (firm f) can match with.

(a) Write down inequalities describing the set of feasible matches.

(b) On the assumption that the Birkhoff-von Neumann Theorem remains true (it does), how does the theory differ? Write down the primal and dual problems for the graph in the figure.