Welfare Economics

1. Here is a version of the First Welfare Theorem with interdependent preferences. Suppose in a pure exchange economy, each individual has a 'private preference order' representable by a utility function $v_i : X_i \to \mathbf{R}$. Suppose that individuals' preferences are aggregates of everyone's private order. That is, for each individual i there is a function $u_i : R^I \to \mathbf{R}$ such that for any two allocations x and y, $x \succ_i y$ iff $u_i(v_1(x_1), \ldots, v_I(x_I))$. Suppose now that private preferences are locally non-satiated, and suppose too that individuals preferences are altruistic in the sense that for all i, $Du_i \gg 0$; that is, i's utility is increasing in his and others private utility. Show that the conclusion of the First Welfare Theorem still holds with respect to private preferences; that is CE is PO with respect to private preferences. Does this imply the first welfare theorem?

2. Fix utility levels u_1 and u_2 for individuals 1 and 2, respectively, and define $U(u_1, u_2)$ to be the set of all aggregate endowments that can be allocated so as to give individual 1 utility at least u_1 and person 2 utility at least u_2. The lower boundary of this set is called the *community indifference curve* for the utility pair (u_1, u_2).

   (a) Let (x^*, p^*) be a competitive equilibrium for an exchange economy with aggregate endowment $e = (e_x, e_y)$. Let u_i denote the utility realized by person i at the competitive equilibrium. Show that if preferences are locally non-satiated, then at at a competitive equilibrium price vector, expenditure on $U(u_1, u_2)$ is minimized at the aggregate endowment.

   (b) Show that if the utility functions of the two individuals is strictly quasi-concave, $U(u_1, u_2)$ is a "strictly convex" set in the sense that the interior of the line segment connecting any two points in the set is in the interior of the set.

   (c) Show that if utility is upper semi-continuous and $f = (f_x, f_y)$ is an endowment bundle that can be allocated to give the consumers $u_i' > u_i$, then at the competitive equilibrium prices, the value of f exceeds the value of e. Such an endowment is called a *potential Pareto improvement* because it *could* be allocated to realize a Pareto improvement, but of course not all allocations of the endowment are Pareto improvements.

   (d) Show that the converse is not true.

   A *project* is a vector $h \in \mathbf{R^2}$ that represents a change in the endowment vector. If the project is run at *scale* s, then the endowment vector will be $e + sh$. A project *passes the net-benefit test* if $p \cdot h > 0$.

   (e) Suppose that utility is C^1 and localy non-satiated. Show that if a project h passes the net-benefit test, then there is an $s^* > 0$ such that for all $0 < s < s^*$, the endowment $e + sh$ is a potential Pareto improvement.

3. Consider an exchange economy in which the endowment allocation ω is not Pareto optimal. Let u_n and v_n denote the direct and indirect utility functions of individual n. Define $R_n = \{(p, m_n) : v_n(p, m_n) \geq w_n(\omega_n)$, and let $R = \{(p, m_1, \ldots, m_N) : (p, m_n) \in$

R_n for all $n\}$. Suppose thatv_n has the Gorman polar form $\alpha_n(p)m+\beta_n(p)$ for all $(p,m) \in R_n$. All functions are as differentiable as you need them to be. Suppose too that all the α_n are identical.

(a) What is individual n's demand function?

(b) Show that aggregate demand on R is independent of the distribution of income.

(c) Suppose that x^* is an allocation Pareto superior to ω, and suppose that it is supported by price vector $p^* \gg 0$. Let

$$P = \{(m_1, \ldots, m_N) : \sum_n m_n = p^* \sum_n \omega_n \text{ and all } v_n(p^*, m_n) \geq u_n(\omega_n)\}.$$

Give an expression for $\sum_n v_(p^*, m_n)$ on P.

(d) Show that the part of the utility possibility frontier corresponding to allocations Pareto superior to ω is a simplex; that is, for some number U, $\sum_n u_n = U$.

4. An exchange economy has two people and three goods. Let x_n denote person 1's consumption of good n, and let y_n denote person 2's consumption of good n. Utility for each consumer is quasi-linear. For person 1:

$$u_1(x,y) = x_1 - \frac{\alpha_1}{2}x_1^2 + x_2 - \frac{\alpha_2}{2}x_2^2 + x_3 - \lambda_1 y_1 - \lambda_2 y_2 - \lambda_3 y_3.$$

For person 2,

$$u_1(x,y) = y_1 - \frac{\beta_1}{2}y_1^2 + y_2 - \frac{\beta_2}{2}y_2^2 + y_3 - \gamma_1 x_1 - \gamma_2 x_2 - \gamma_3 x_3.$$

Notice that there are externalities in consumption. The endowment for consumer 1 is $(1, 0, e)$, and for consumer 2 it is $(0, 1, f)$.

There is a sales tax on goods 1 and 2. The seller of good n receives price p_n, while the buyer pays price $p_n + t_n$. Tax revenues split equally and handed back to the consumers as a lump-sum transfer. (That is, in deciding how much of a commodity to buy, the consumer does not account for the return of some share of the cost through the tax split.)

(a) What is each consumer's budget constraint?

(b) Define a sensible notion of competitive equilibrium for this economy.

(c) Compute the competitive equilibrium for arbitrary (small) taxes t_1 and t_2, assuming the allocation is interior.

(d) Compute the derivative of both individuals' utilities with respect to the tax rates.

(e) Are there small but non-zero taxes that would give an equilibrium allocation Pareto-better than that achieved at $t_1 = t_2 = 0$?

5. An allocation x^* in an exchange economy is a *weak Pareto optimum* if there is no feasible allocation x' such that each individual is better off under x' than under x'. State and prove a theorem about the relationship between competitive equilibria and weak Pareto optima.

6. Consider an exchange economy with L locally non-satiated individuals and N goods, and an aggregate endowment $\omega \gg 0$. Individual i is *resource related* to household j if at every feasible allocation x there is another allocation x' and a resource vector z such that

   - x' is at least as good as x for all individuals k, and strictly better for individual j,
   - x' is feasible for the economy with aggregate endowment ω',
   - $\omega' > \omega$, and $\omega'_k = \omega_k$ for all $k \neq i$.

   Individual i is indirectly resource related to j if there is a sequence of individuals, $i = k_1, k_2, \ldots, k_m = j$ such that each individual k_j is directly resource related to her successor k_{j+1}.

   (a) Say in words the meanings of resource relatedness and indirect resource relatedness.

   (b) Show that if i is resource-related to j and (p^*, x^*, y^*) is a compensated equilibrium, and if $p^* x_j^* > 0$, then $p^* x_i^* > 0$.

   (c) Use the conclusion of part 1 to conclude that if i is indirectly resource-related to j and (p^*, x^*, y^*) is a compensated equilibrium, and if $p^* x_j^* > 0$, then $p^* x_i^* > 0$.

   (d) Show that if every individual is indirectly resource-related to other individual, every quasi-equilibrium is a competitive equilibrium.

7. Construct a one-consumer one-firm economy in which the production set is convex, preferences are continuous and convex, and there is nevertheless a Pareto-optimal allocation which cannot be supported as a quasi-equilibrium. What assumptions of the 2nd welfare theorem does your example violate?

8. Suppose that two goods are each made with labor and capital, and that the production functions for each good satisfy the usual neoclassical assumptions: they are C^2 and concave. Give a characterization of the rate that one good can be transformed into another.