Problem Set on Zero-Sum Games

1. A two person zero-sum game is described by a single matrix M; m_{ij} is the payoff to the row player of choosing strategy i when the column player chooses strategy j. Since the game is zero-sum, the payoff to the column player of the i, j pair is $-m_{ij}$.

   (a) Suppose column chooses mixed strategy q. What is the vector of payoffs against each pure strategy i that column could choose? What is the reward to row of choosing mixed strategy p? How much does column get?

   (b) Suppose that column's goal is to give row as little utility as possible. Row's *security level* is the maximal amount of utility that row can guarantee himself by a suitable strategy choice no matter what column does after seeing row's strategy choice. Formulate the problem of finding row's security level as a linear program. The solution should give you both a value for the row player and a strategy p^*.

   (c) Write down and interpret the dual of your linear program. The dual should give you both a value w^* and a strategy q^*.

   (d) The fundamental theorem of two-person zero sum games, due to von Neumann in 1927, is that finite games have a value, that is, a security value v^*, and a (p^*, q^*) pair such that p maximizes row's security value, q maximizes column's security value, and $p^*Mq^* = v^*$. What is the connection between (p^*, q^*) and the linear programs of parts b) and c)?

   (e) Suppose we think of the (p^*, q^*) pair as a solution concept for this class of games. How is this solution related to Nash equilibrium?

   (f) Describe some properties of the solution set to a given game. Compare to Nash equilibrium in other kinds of finite games.